Comparing Direct and Indirect Seasonal Adjustments of Aggregate Series

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Different estimates of the seasonal effects in an economic time series can arise from different choices of the options in the seasonal adjustment software. Or, with an aggregate series, different estimates can also result from different choices of the level of aggregation at which seasonal adjustment is performed. One purpose of this paper is to illustrate some issues that arise in the seasonal adjustment of aggregate series. A second focus is to describe some useful ways of detecting adjustment inadequacies and assessing the quality of the adjustments.

We will discuss what makes an acceptable seasonal adjustment and how to look for signs of inadequacy in the adjustment, particularly with aggregate series. We will also discuss briefly some diagnostics for judging the quality of the adjustment. We will show why it may not be useful to analyze the ratio or difference of the direct and indirect adjustments.

1. Definitions

Seasonal adjustment is the process of estimating and removing the seasonal effects from a time series. Seasonal factors are obtained from procedures that decompose the time series into seasonal, trend-cycle, and irregular components. Most economic time series are decomposed using a multiplicative decomposition, so the seasonal factors are divided out of the original series. An additive adjustment is appropriate for some series, in which case estimated seasonal factors are subtracted from the original series.

The most fundamental requirement of a seasonal adjustment, regarding quality, is that there be no estimable seasonal effect still present in the seasonally adjusted series. The presence of estimable seasonal effects in either the seasonally adjusted series or the detrended seasonally adjusted series (i.e, in the irregular component) is generally what is referred to as *residual seasonality*.

When appropriate, seasonal adjustment also includes adjustment for other largely predictable calendar effects, most often *trading day effects*, which are effects due to weekly activity cycles. We will not discuss trading day effects and their estimation in this paper.

When a time series is a sum (or other composite) of a set of series in which each series is seasonally adjusted, we can sum the seasonally adjusted component series to get a seasonally adjusted aggregate series. This kind of adjustment is called an *indirect adjustment* of the aggregate series. The alternative is the *direct adjustment* obtained by applying the seasonal adjustment procedure directly to the aggregate data. For example, when we seasonally adjust export series at the individual end-use-code level and then sum the adjustments to get Total Exports, we have an indirect adjustment of Total Exports. If we sum the individual series first to get Total Exports and then seasonally adjust the total, we have a direct adjustment of Total Exports.

Under most circumstances, the direct and indirect adjustments for an aggregate series are not identical. There are some very limited situations in which the direct and indirect adjustments could coincide, particularly if the adjustments are additive. For a multiplicative decomposition, the conditions required for identical adjustments are even more restrictive. For more information, see Pfefferman, Salama, and Ben-Turvia (1984).

2. Diagnostics for Direct and Indirect Adjustments

Whether or not direct or indirect adjustment is better for a given set of series depends on the set of series in question (Dagum, 1979, and Pfefferman et al, 1984). Generally speaking, when the component series that make up the aggregate series have quite distinct seasonal patterns and have adjustments of good quality, indirect seasonal adjustment is usually of better quality than the direct adjustment. However, when the component series are noisy but have similar seasonal patterns, then summing the series may result in noise cancellation, and the direct seasonal adjustment is usually of better quality than the indirect adjustment. In other situations, it is not clear a priori which adjustment will be better.

How do we know if it's better to use direct or indirect adjustment for a given set of series? How similar do the component series need to be in order for direct adjustments to be of superior quality? If the component adjustments are acceptable, will the indirect adjustment always be acceptable as well? The best way to answer these questions is to look at diagnostics.

The Census Bureau uses its X-12-ARIMA software to produce seasonally adjusted numbers. X-12-ARIMA and its predecessors, X-11 and Statistics Canada's X-11-ARIMA, are widely-used seasonal adjustment programs. One of the major improvements of X-12-ARIMA is its additional diagnostics.

2.1 Features of a Quality Adjustment

As mentioned above, a lack of residual seasonality is the most fundamental requirement of a good quality seasonal adjustment. Other important qualities of a good adjustment are lack of bias in the level of the series and the stability of the estimates. Lack of bias in the level means that the local level of the series will be similar for both the original series and the seasonally adjusted series. Stability of the estimates means that as new data are added and incorporated into the estimation procedure, the revisions to the past estimates are small. Large revisions can indicate that the original estimates are misleading or even meaningless.

There are other features that may be desirable in an adjustment. Some users may prefer a smoother adjustment to aid in detecting turning points. However, it is important to remember that achieving such desired features can conflict with the quality requirements mentioned earlier. For example, the smoother of two adjustments may also be the

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one that is more susceptible to large revisions as future data become available and are included in the adjustment calculations. Some users may find an adjustment with large revisions to be unsuitable. Therefore, it is important to balance all the qualities and features desired.

It is important to always check the quality of the adjustment. This applies to the direct and indirect adjustments, as well as the adjustments of all the component series. We will give examples later in the paper to show the importance of diagnostics.

2.2 Diagnostics for Residual Seasonality

One of the most important diagnostics is the spectral diagnostic for residual seasonality and trading day, examples of which will be given in this paper. For series that are long enough, the Census Bureau encourages users to look at this diagnostic to see if there are any residual calendar effects.

The spectrum of an observed time series shows the strength, or amplitude, of each frequency component when the data are decomposed into such components. For a monthly series with a strong seasonal effect, the spectrum will have especially large amplitudes at the frequencies associated with components that repeat every year, i.e., every twelve months, every sixth months, every three months, etc. Therefore, for monthly series with a strong seasonal effect, we will see peaks in the spectrum at frequencies $\boldsymbol{k}/12$ cycles per month, for $1 \leq \boldsymbol{k} \leq 6$. For a quarterly series with a strong seasonal effect, we will see peaks in the spectrum at the frequencies ¼ cycle per quarter and at ½ cycle per quarter. In X-12-ARIMA, spectral peaks are measured in units called "stars" with six stars being labeled as "visually significant." For more information, see Findley, Monsell, Bell, Otto, and Chen (1998) and Soukup and Findley (1999).

With inadequate seasonal adjustments that contain residual seasonality, the residual effects are usually rather weak, and it is necessary to remove any very strong frequency components from the adjusted series before the spectrum calculation to enable the spectrum to reveal the presence of the seasonal component. If a series has strong long-term trend movements, the low frequencies associated with the long-term trend movements will have amplitudes that dominate the spectrum. Since the irregular component of the seasonal adjustment decomposition is the detrended seasonally adjusted series, X-12-ARIMA plots the spectrum of the irregulars to help the user detect residual seasonality.

There are also some other statistical procedures, including F tests, that can be used to detect residual seasonality, but for series that are long enough, the spectral graph is the most sensitive diagnostic to test for residual seasonality.

2.3 Additional Diagnostics

Two different adjustments can both be successful in the sense that their adjusted series have no residual seasonality but one may be more attractive to some users but less attractive to others. For this reason, we believe that once we have determined there is no residual seasonality, it is important to look at additional diagnostics.

X-12-ARIMA's stability diagnostics are the sliding spans and revision history diagnostics. X-12-ARIMA also contains diagnostics for month-to-month (or quarter-to-quarter) percent changes to compare the smoothness of two adjustments. X-12-ARIMA also computes smoothness measures for comparing the direct and indirect adjustment as introduced by X-11-ARIMA.

Seasonal adjustments for any given month will change as new data are introduced into the series. The sliding spans diagnostic computes separate seasonal adjustments for up to four overlapping subspans of the series. For more infomation on sliding spans diagnostics, please see Findley, Monsell, Shulman, and Pugh (1990), Findley et al (1998), or the *X-12-ARIMA Reference Manual* (2001).

Another way to look at the revisions for a series is to compare the initial adjustment for any given point (the adjustment when that particular point is the latest point in the time series) to the final adjustment for that point (the adjustment when all the data in the time series is included in the adjustment). For more information on the revision diagnostics, see Findley et al (1998) or the *X-12-ARIMA Reference Manual* (2001). For more information on graphs of the revision diagnostics, see Hood (2001). We will show some examples of revision history graphs later in this paper.

X-12-ARIMA also contains the M and Q quality diagnostics developed at Statistics Canada and included in X-11-ARIMA. These diagnostics are a set of eleven numbers that help the users see possible problems in the quality of the adjustments. There were designed so that any number greater than 1.0 signals a possible problem. X-11-ARIMA and X-12-ARIMA provide M's and Q's for both the direct and indirect adjustment. While these numbers are useful in helping the user see some potential problems with the adjustments, they were not designed to be used to choose between the direct and indirect adjustment. In other words, if the M's and Q's are acceptable for both the direct and indirect adjustment, we should not prefer one adjustment because is has smaller M and Q values.

Next we will look at some examples showing how we use diagnostics available in X-12-ARIMA.

2.4 Example–Residual Seasonality in the Aggregate Series

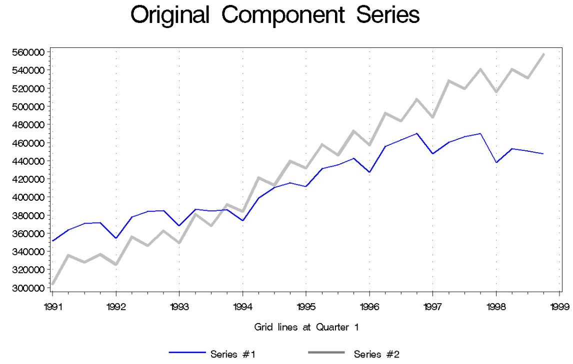
If the component series have no residual seasonality, are we guaranteed that the indirect adjustment will have no residual seasonality? We will demonstrate that it is possible to have two series with no apparent residual seasonality, and yet still have residual seasonality in the aggregate series.

The use of inappropriate options in seasonal adjustment software can result in bad adjustments. Options that were chosen in years past for a series may be suboptimal or inappropriate for the series now. And when we add together several adjusted series produced from suboptimal options, we can sometimes see residual seasonality in the aggregate series. This is why, at the Census Bureau, we believe it is important to check the seasonal adjustment diagnostics every year, including the diagnostics for the aggregate series as well as for the component series. Other agencies also stress the importance of diagnostics for the direct and indirect adjustment (Cannon, 2000).

For the purpose of an example, we have selected a simple aggregate series. The two composite series are real series, and the Census Bureau publishes the aggregate series, but not in the way described below. For the published series, all the diagnostics have been checked carefully. We chose seasonal filter lengths for one series that were inappropriate given the diagnostics from the series. Using these filters, it is possible to create an aggregate series with residual seasonality in the indirect adjustment.

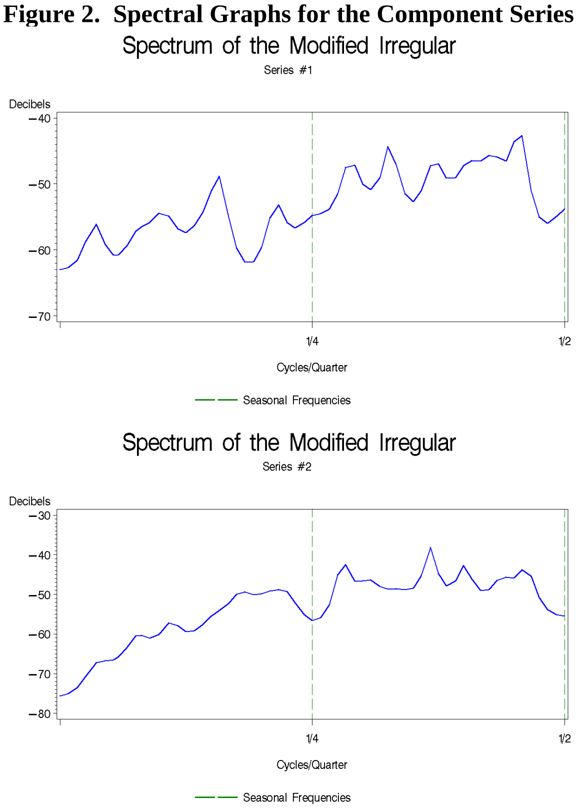
The last eight years of the two original (unadjusted) composite series are shown in Figure 1.

Figure 1. Graph of the Two Components



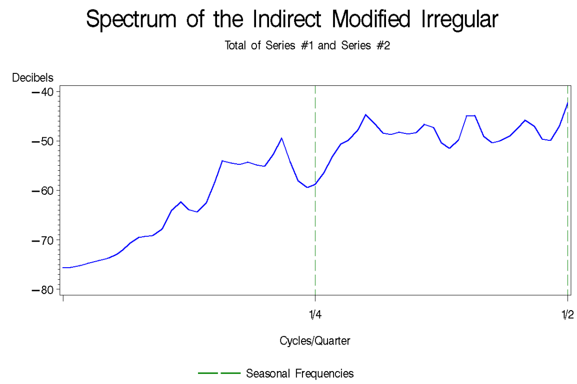
The spectral graphs for the components are shown in Figure 2.

Figure 2. Spectral Graphs for the Component Series



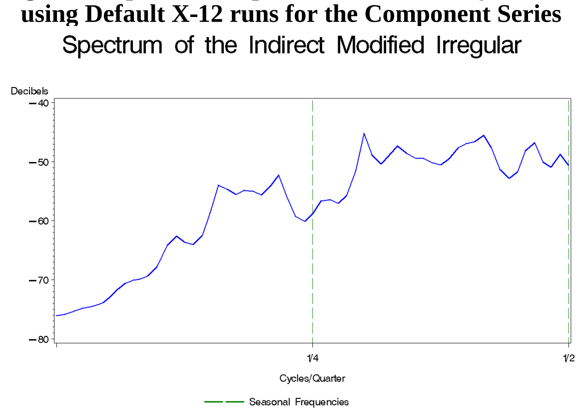
We would expect that seasonal frequencies at ¼ and ½ would have been suppressed in the spectrum of the seasonally adjusted series. In the first graph in Figure 2, there is a slight seasonal peak on the right of the graph at ½. However, the peak is not marked as "visually significant" by X-12-ARIMA, nor is it the dominant peak of the graph. Therefore, there is no strong signal of residual seasonality for this series alone. The spectral graph for the total of the two series, shown in Figure 3, has a peak on the right side of the graph at the frequency ½ of a cycle per quarter. This peak is marked by X-12-ARIMA as "visually significant" and is the highest peak on the graph, signaling residual seasonality.

Figure 3. Spectral Graph of the Indirect Adjustment for the Total



This adjustment is different from the indirect adjustment obtained from default runs of X-12-ARIMA for the component series. The spectral graph for the default runs of the component series is shown in Figure 4. There are no seasonal peaks whatsoever.

Figure 4. Spectral Graph of the Indirect Adjustment, using Default X-12 runs for the Component Series



2.5 Example–Revisions and Smoothness in the Aggregate Series

The Census Bureau publishes US Single-Family Housing Starts for four regions of the United States: Northeast, Midwest, South, and West. All four series have somewhat similar seasonal patterns in that housing starts are higher in the summer and lower in the winter. Yet the drop in housing starts in the winter months is more pronounced in the Northeast and Midwest regions than in the South and West regions. Are the seasonal patterns similar enough for the direct adjustment to be of better quality than the indirect adjustment? We will look at some diagnostics.

We first checked the direct and indirect seasonal adjustments for residual seasonality. Both adjustments were found to be acceptable, so the next step is to make some decisions based on revisions.

The direct adjustment, initial and final, is shown in Figure 5, and the indirect adjustment, initial and final, is shown in Figure 6. The final seasonal adjustment is plotted with the solid line. Initial estimates for the adjustment are shown as the dots. We want the adjustment with smaller revisions, i.e., with initial adjustments that are closer to the final adjustment. Both the direct and indirect adjustments have some months at which there are large revisions—February and October of 1997 are two examples. However, there are more months with large revisions for the direct adjustment—see for example early 1996, December 1996, and November and December 1997. We prefer the indirect adjustment for US Total Single Family Housing Starts because of the smaller revisions.

Figure 5. Revisions, Initial to Final, for the Direct Adjustment for US Housing Starts

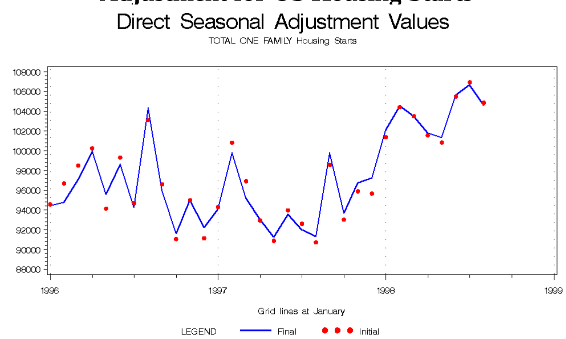
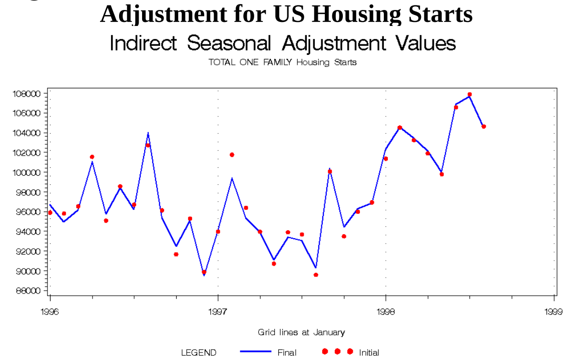


Figure 6. Revisions, Initial to Final, for the Indirect Adjustment for US Housing Starts



3. Features of the Ratio and Difference of Two Adjustments of the Same Series

Now that we have looked at some ways to measure the adequacy and quality of aggregate adjustments, we will look at the difficulties involved in trying to compare the direct and indirect adjustments by computing ratios or differences.

When comparing two adjustments of the same series by looking at the series of their numerical ratios or differences, seasonal adjusters and data users sometimes see seasonal patterns. We will explain how this can happen even with successful seasonal adjustments.

3.1 Ratios and Differences

In economic time series, it is a very common phenomenon that the variations in the series increase as the level of the series increases. When we seasonally adjust series with level-dependent variability, we use a multiplicative decomposition model for seasonal adjustment. To compare multiplicative adjustments, it is more natural to look at ratios rather than their differences. With series that are adjusted additively, it is more natural to look at differences.

3.2 Apparent Seasonality in the Ratios and Differences

It is simple to show algebraically that residual seasonality in the ratio (and the difference if the adjustment is additive) of two adjustments of the same series can be a natural occurrence and not necessarily an indication of a problem in either adjustment. If you have two adjustments of the same series, and if the seasonal factors of both adjustments show very little evolution over time, then the ratio of the two adjusted series will be seasonal.

We will focus on monthly adjustments in the equations. Similar equations also hold for quarterly series.

Let Y_t be the original series. Let $S_t^{(1)}$ be one series of seasonal factor estimates. Let $S_t^{(2)}$ be a second series of seasonal factor estimates. In the case of direct and indirect adjustment, one set of seasonal factors will be the indirect seasonal factors obtained by dividing the original values by the adjusted values from the indirect adjustment.

For multiplicative adjustment, the adjusted series, A_t, is the original series divided by the seasonal factor estimates. So from the two series of seasonal factors we have two different adjustments:

$$A_t^{(1)} = \frac{Y_t}{S_t^{(1)}} \quad \text{and} \quad A_t^{(2)} = \frac{Y_t}{S_t^{(2)}}.$$

If both seasonal factor estimates are periodic, i.e., $S_{t-12}^{(1)} \approx S_t^{(1)}$ and $S_{t-12}^{(2)} \approx S_t^{(2)}$ for all t, then the ratio will be periodic also, and we will see a seasonal pattern in the ratio. From

$$\frac{A_t^{(1)}}{A_t^{(2)}} = \frac{Y_t/S_t^{(1)}}{Y_t/S_t^{(2)}} = \frac{S_t^{(2)}}{S_t^{(1)}}. \qquad (1)$$

we obtain $A_{t-12}^{(1)} \,/\, A_{t-12}^{(2)} \approx A_t^{(1)} \,/\, A_t^{(2)}$.

Let's look at a conceptual example. Let's suppose the seasonal factor for the first adjustment, $S_t^{(1)}$, is 1.073 for January in the most recent year, telling us the original unadjusted quarter one numbers should be decreased by 7.3%. Let's also assume that the estimates of the seasonal factors for January are reasonably stable ($S_{t-12}^{(1)} \approx S_t^{(1)}$), so that the estimates for the first quarter of every year are approximately 1.073. Let's say that the seasonal factor for the second adjustment, $S_t^{(2)}$, is 1.064 for January in the most recent year and the estimates of the seasonal factors are stable. Therefore, when we divide both seasonal factors into the same original series, the ratio of the two seasonal factors (see equation (1) above) for every first quarter is 1.064/1.073 = 0.992. If the same kind of stability is found in the monthly estimates for the other months, then we have a series of ratios that are periodic and will observe a seasonal pattern in the ratio.

Keep in mind that if we take the ratio of two seasonal adjustments of the same series, the ratio most likely will be close to 1. Also remember that any seasonal adjustment is an estimate, and therefore imprecise. Movements in the ratio of the two adjustments can be smaller in magnitude than the uncertainty surrounding each of the seasonal adjustments. Thus, in the example above, if possible error in the seasonal factor estimates or seasonally adjusted data is close to one percent, then the ratio 0.992 will not be statistically different from 1.0.

For additive adjustment, the basic principles are the same. The adjusted series A_t is the original series minus the seasonal factor estimates. To compare the two adjustments, we would take differences instead of ratios.

$$A_t^{(1)} - A_t^{(2)} = (Y_t - S_t^{(1)}) - (Y_t - S_t^{(2)}) = S_t^{(2)} - S_t^{(1)}$$

If both seasonal factor estimates are periodic, then the difference of the seasonal factors will also be periodic and therefore seasonal.

3.3 Example–Direct/Indirect Ratio for Total Exports

We will compare a default X-12-ARIMA direct adjustment to an indirect adjustment for US Total Exports.

3.3.1 Indirect Adjustment for Total Exports

We looked at the spectral diagnostics in X-12-ARIMA for evidence of seasonality. The spectral graphs for the original, unadjusted series and for the seasonally adjusted series are shown in Figures 7 and 8.

Notice that the spectrum of the original series in Figure 7 has a somewhat broad peak at both seasonal frequencies: ¼ cycle per year and ½ cycle per year. This is an indication of changing seasonality present in the original series.

Figure 7. Spectrum of the Original Series, Quarterly Total Exports

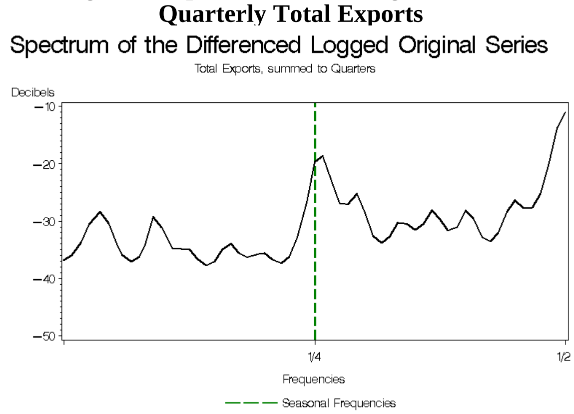
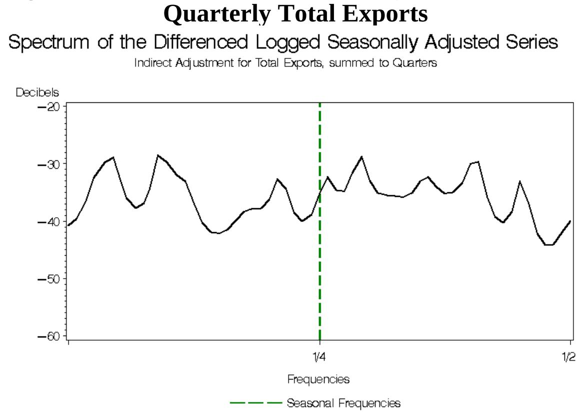


Figure 8. Spectrum of the Seasonally Adjusted Series, Quarterly Total Exports



Notice that peaks at the seasonal frequencies are suppressed by the seasonal adjustment as shown in Figure 8. Though there is still a small peak at ½, it is not identified by X-12-ARIMA as visually significant. Therefore, we can conclude there is no estimable seasonal effect still present in the seasonally adjusted series.

3.3.2 Direct Adjustment for Total Exports

The default X-12-ARIMA run for the direct adjustment of Total Exports has a spectrum (not shown here) which has no sign of residual seasonality.

3.3.3 Direct/Indirect Ratio

Figure 9 shows, in a year over year graph, the ratio of the direct and indirect adjustment for Total Exports. We can see the seasonality in the ratio. Generally, we see a first quarter to second quarter increase, a third quarter to fourth quarter decrease, and a fourth quarter to first quarter increase. The X-12-ARIMA diagnostics show signs of estimable seasonality, among them, the F-test for stable seasonality was 33.1. The spectral graph shown in Figure 10 also shows signs of moving seasonality in the ratio—broad peaks around ¼ and ½.

Note that the range of the ratios is very small in magnitude compared to the original series.

Figure 9. Year Over Year Graph, Ratio between the Direct and Indirect Adjustments

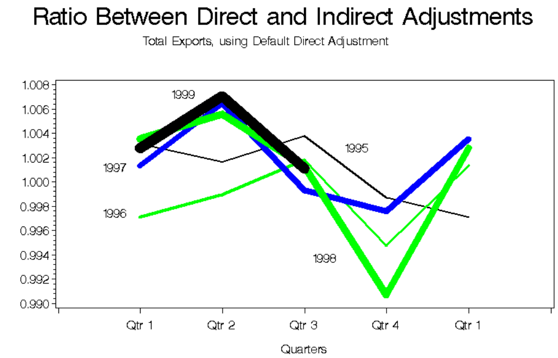
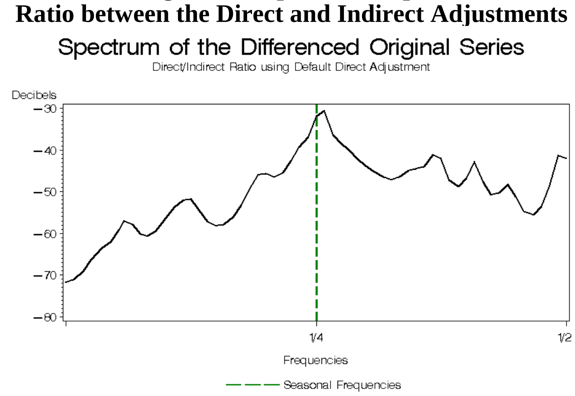


Figure 10. Spectral Graph, Ratio between the Direct and Indirect Adjustments



Will we always find the phenomenon that diagnostics show possible "residual seasonality" in the ratio? We can very easily change the seasonal filter lengths for the direct adjustment. If we change the filter lengths used by X-12-ARIMA for some of the quarters in the direct adjustment of Total Exports, we can change the ratio between the direct and indirect adjustment.

Notice in Figure 11 below that the ratio has changed from the ratio shown in Figure 9. Notice also that the range of the ratios has been reduced. The X-12-ARIMA diagnostics show fewer signs of estimable seasonality. The F-test for stable seasonality was 12.6. The spectral graph in Figure 12 also shows signs of reduced estimable seasonality in the ratio. Now there is not a peak at the frequency ½ and a lower, flatter peak at ¼.

Figure 11. Year Over Year Graph, Ratio between the Direct and Indirect Adjustments, Modified Direct Adjustment

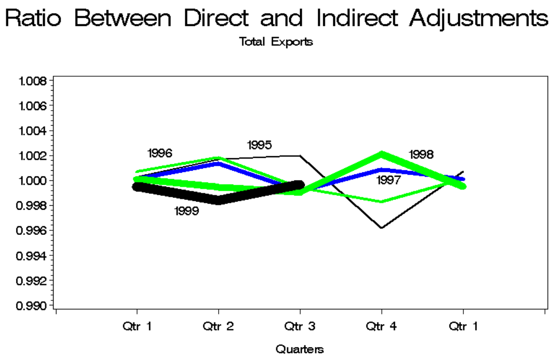
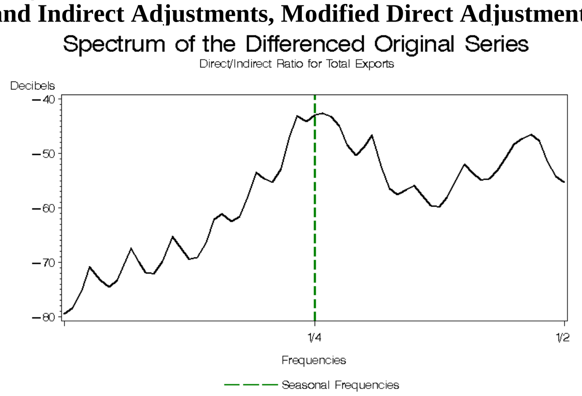


Figure 12. Spectral Graph, Ratio between the Direct and Indirect Adjustments, Modified Direct Adjustment



4. Conclusions

Diagnostics for the indirect adjustment of the aggregate can be very helpful in determining the best options for the component series.

When two competing estimates of the seasonal factors of a time series are both rather stable, in the sense that each calendar month's (or calendar quarter's) factor changes little from one year to the next, then the factors from the two adjustments will differ in a consistent way. In this case, the ratio of the two multiplicative seasonal adjustments will necessarily have a seasonal component.

A basic problem with ratios of adjusted series is that their movements are often on the same scale as the "noise" level in the series, in which case the movements can fail to have statistical significance.

5. Acknowledgments

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